Relation Of Height, Width And Depth Of The Mandible*

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INTRODUCTION

Interpretation of morphological patterns within the wide variation of the human face has intrigued anthropologists (Goldstein, 1936; Meredith, 1963), and orthodontists (Hellman, 1939; Björk, 1947; Downs 1948). Meredith (1960) observed that research on form of the head and face has progressed rather slowly. Graphic analysis and calculation of size parameters have been reported for mandibular body length by Nanda (1955) and Meredith (1961), mandibular maximum length by Harris (1962), and bigonial width by Newman and Meredith (1956). Harvold (1963) studied relationship of maximum mandibular length to growth rate and to a maxillary dimension. Correlation of mandibular body length to bigonial width growth rate was reported by Meredith (1963). These investigators have studied (a) size of mandibular dimensions independently and (b) joint growth rate of only two mandibular dimensions. There has been no cephalometric research on size interrelations of all mandibular dimensions.

Methods for obtaining accurate measurements of individual bones have not been available until recently (Savara, 1965). Morphology of the growing human face has been studied qualitatively (Brodie, 1941), but anthropometrists have been reluctant to utilize advances in statistical techniques for multiple measurements (Garn, 1955; Schull, 1962).

This paper presents the findings of a longitudinal study on the interrelations of five dimensions of the mandible, representing one measure of height, and two each of width and depth. The results are presented for predicting each of the five dimensions from a combination of the other four.

MATERIALS AND METHODS

The sample consisted of twenty-seven girls enrolled in Child Study Clinic for the longitudinal study. They were of northwest European ancestry and of middle socioeconomic status. The girls were examined annually within one month of each birthday and had complete records from four through nine years.

The five mandibular dimensions were measured on posteroanterior, lateral, and lateral open-mouth cephalograms derived from the serial records of each child. The cephalograms were taken with the child's head oriented to the Frankfort plane in a Broadbent-Bolton cephalometer. Landmarks (Figure 1) were defined as follows:

Right and left gonion: midpoint on curve formed by intersection of posterior border of the ramus and inferior border of the body of the mandible in the gonial area on lateral cephalogram and maximum width of the mandible in gonial area on lateral surface on posteroanterior cephalogram.

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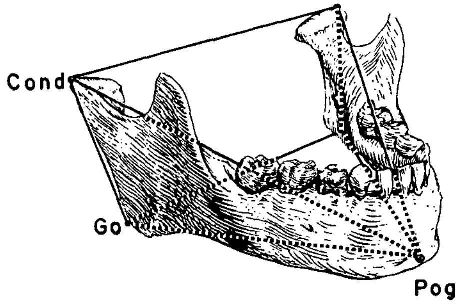

Figure 1 Illustration of mandible with landmarks and dimensions measured.

Pogonion: most anterior point on the arc formed by the intersection of the inferior border of mandible and the anterior surface of the mandible on lateral cephalogram; and midline of body of mandible on posteroanterior cephalogram.

Right and left condylion: most superior point on the outline of the condyle on lateral cephalogram, and point of intersection of the extended line drawn along the lateral surface of the ramus and midportion plane on posteroanterior cephalogram.

Landmarks were marked for each at one sitting on the entire series of cephalograms as described by Newman and Meredith (1956). Each landmark was examined first on the lateral cephalograms for a given subject and the age-to-age location of the landmark was determined. The point was then marked on acetate tracing paper with a fine-pointed lead. After one landmark had been located on all films of a series, the same procedure was followed for each of the other landmarks. Similarly, landmarks were located on the frontal cephalograms.

Landmarks were measured to 0.1 mm accuracy in two coordinates on the lateral tracing and one coordinate on the frontal tracing using an Oscar Analog Reader and Decimal Converter. An exclusive Fortran Program was written for an IBM 7094 computer and utilized for correction of coordinates and computation of distances. The coordinates were corrected for distortion and enlargement and distances between the landmarks for each dimension computed. Details of the method for three-dimensional correction of landmarks and computation of distances have been given by Savara (1965) and are not repeated here.

Measurements obtained at each age and their symbol designations were:

- x_1 *Ramus Height* from right condylion to right gonion and left condylion to left gonion
- x_2 *Body Length* from right gonion to pogonion and left gonion to pogonion
- x_3 *Maximum Length* from right condylion to pogonion and left condylion to pogonion
- x_4 *Bigonial Width* from right gonion to left gonion
- x_5 *Bicondylar Width* from right condylion to left condylion

In this study, ramus height, body length and maximum length represented the average of right and left measurements, while bigonial width and bicondylar width represented a single measurement.

The statistical approaches utilized in this study were those of simple and partial correlation, and multiple regression analyses. The correlation coefficient, "r", expresses in one statistic the relation of one variable to a second variable. Thus the simple correlation of ramus height (x_1) with body length (x_2) is calculated without considering variability that exists in other parts of the mandible. This is the usual type of correlation analysis encountered in the orthodontic literature. However, if x_1 is correlated with x_2, while at the same time the variation of maximum length (x_3), bigonial width (x_4) and bicondylar width (x_5) are "statistically controlled", a partial correlation coefficient is computed. "Statistical control" implies that, during this computation, the maximum length (x_3), bigonial width (x_4) and bicondylar width (x_5) values were adjusted simultaneously to make them effectively equal (i.e., each variable was fixed at a certain level). Thus by partial correlation, the "true correlation" between two variables is made apparent after the combined effects of other variables have been removed.

Simple linear regression is the straight-line relation between two variables in which one acts as the predictor ("independent") and the other is predicted ("dependent"). For instance, to predict ramus height from body length, ramus height is utilized as the dependent variable and body length as the independent variable. The regression equation can be expressed in the form:

$$x_1 = A + B_2x_2$$

where x_1 is ramus height, x_2 is body length, and A and B_2 are the regression coefficients. An extension of the idea of simple linear regression is to consider the regression of one dependent variable on several independent variables, e.g., ramus height on body length, maximum length, bigonial width and bicondylar width. This relation can be expressed in a multiple regression equation of the form:

$$x_1 = A + B_2x_2 + B_3x_3 + B_4x_4 + B_5x_5$$

where x_1 is ramus height, x_2 is body length, x_3 is maximum length, x_4 is bigonial width, and x_5 is bicondylar width; A, B_2, B_3, B_4, and B_5 are the regression coefficients. Multiple regression is very useful since the prediction of x_1 from any single independent variable is often not as reliable as the prediction from a combination of several independent variables.

A data analysis program which could handle a multivariate population of five variables was developed. For data analysis at each age, the program was divided into four phases:

Phase 1 permitted a standard normal transformation of the raw data of all variables for determination of distribution properties (untransformed data was used in all other phases of analysis).

Phase 2 provided for entering the raw data directly and computed means, standard deviations and coefficients of variation.

Phase 3 computed simple and partial correlation coefficients for all possible combinations of variables.

Phase 4 provided for removal of extremely large or small observations (outliers) from the data and then computed five multiple regression equations for each age, one with each variable as the *dependent* variable and the remaining four as *independent* variables.

Results

The raw data of each variable were processed via Phase 1. The Chi Square tests of the sample frequency distributions were found to support the hypothesis that all variables followed the normal distribution.

The means, standard deviations and coefficients of variation, which were computed in Phase 2, are shown in Table 1. Inspection of the table indicates that the age-to-age variation of one dimension was less than the dimension-to-dimension variation at any age. For instance, the age-to-age variations from four to five years for any dimension were half as large as the dimension-to-dimension variations at four years. Further examination shows that this difference increased with age. Also, Table 1 illustrates that the absolute variation of all dimensions tended to increase with age, and that variability was least for ramus height and body length, intermediate for bigonial width

TABLE 1. CHARACTERISTIC STATISTICS

Mean, Standard Deviation, Coefficient of Variation

Age	Ramus Height			Body Length			Maximum Length			Bigonial Width			Bicondylar Width		
	$\overline{X}$	S.D.	C.V.	$\overline{X}$	S.D.	C.V.	$\overline{X}$	S.D.	C.V.	$\overline{X}$	S.D.	C.V.	$\overline{X}$	S.D.	C.V.
4	4.10	.22	5.4	6.42	.23	3.6	9.37	.31	3.3	7.21	.33	4.6	8.80	.44	5.0
5	4.27	.22	5.2	6.68	.22	3.3	9.70	.28	2.9	7.45	.36	4.8	9.01	.48	5.3
6	4.42	.26	5.9	6.93	.25	3.6	10.01	.32	3.2	7.71	.34	4.4	9.27	.47	5.1
7	4.53	.24	5.3	7.17	.26	3.6	10.31	.32	3.1	7.91	.34	4.3	9.40	.53	5.6
8	4.64	.24	5.2	7.37	.24	3.3	10.55	.32	3.0	8.10	.36	4.4	9.58	.52	5.4
9	4.82	.27	5.6	7.55	.26	3.4	10.81	.32	3.0	8.27	.37	4.5	9.74	.60	6.2

TABLE 2.

SIMPLE CORRELATION COEFFICIENTS BETWEEN EACH DIMENSION AND OTHER MANDIBULAR DIMENSIONS AT EVERY AGE 4 TO 9 YEARS

	Age	Body Length	Maximum Length	Bigonial Width	Bicondylar Width
Ramus Height	4	.36	.65*	.07	–.02
	5	.19	.50*	–.09	.09
	6	.20	.57*	–.11	.14
	7	.02	.41*	–.23	.19
	8	.14	.39*	.03	.03
	9	–.10	.36	–.21	.15
Body Length	4		.85*	.36	.04
	5		.78*	.44*	–.06
	6		.75*	.43*	.12
	7		.74*	.48*	–.08
	8		.70*	.41*	.17
	9		.66*	.51*	–.02
Maximum Length	4			.19	–.01
	5			.24	.00
	6			.13	.17
	7			.18	.13
	8			.19	.19
	9			.16	.18
Bigonial Width	4				–.02
	5				–.04
	6				–.15
	7				–.20
	8				–.12
	9				–.22

*Significant at 5% probability level

TABLE 3.

PARTIAL CORRELATION COEFFICIENTS BETWEEN EACH DIMENSION AND OTHER MANDIBULAR DIMENSIONS AT EVERY AGE 4 TO 9 YEARS

	Age	Body Length	Maximum Length	Bigonial Width	Bicondylar Width
Ramus Height	4	–.50*	.71*	.14	.05
	5	–.29	.54*	–.10	.08
	6	–.36	.62*	.00	.05
	7	–.34	.53*	–.14	.03
	8	–.20	.42*	.03	–.04
	9	–.42*	.55*	–.02	.00
Body Length	4		.88*	.41*	.12
	5		.78*	.38	–.06
	6		.78*	.48*	.11
	7		.79*	.40*	–.15
	8		.69*	.41*	.12
	9		.75*	.47*	–.05
Maximum Length	4			–.27	–.11
	5			–.11	.02
	6			–.24	.00
	7			–.14	.19
	8			–.14	.08
	9			–.18	.17
Bigonial Width	4				–.07
	5				.01
	6				–.20
	7				–.11
	8				–.20
	9				–.19

*Significant at 5% probability level

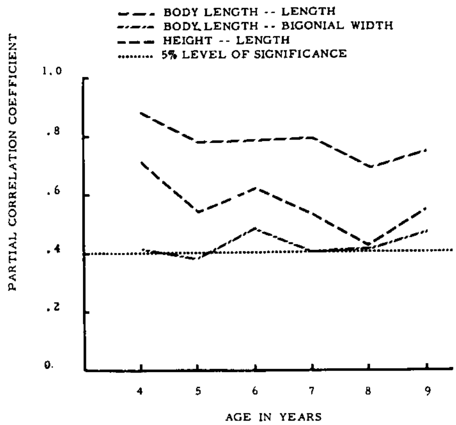

Figure 2 Plot of partial correlations for body length with maximum length and bigonial width, and ramus height with maximum length.

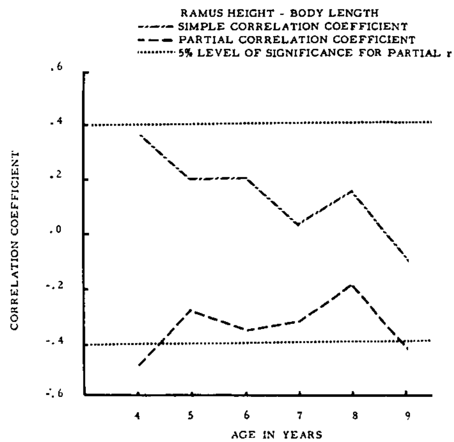

Figure 3 Plot of simple and partial correlations for ramus height with body length.

and maximum length, and greatest for bicondylar width. The corresponding relative variability was low for all dimensions, ranging from three to six percent.

Simple and partial correlation coefficients, computed in Phase 3 and shown in Tables 2 and 3, indicated three statistically significant correlations: maximum length with body length and with ramus height, and bigonial width with body length. Partial correlation coefficients for these variables plotted against age (Figure 2) illustrated a trend that maximum length with ramus height and with body length decreases with age, and that body length with bigonial width increases with age. Tables 2 and 3 depict statistically nonsignificant correlations for the relation of bigonial width to ramus height or maximum length, and for the relation of bicondylar width with any other mandibular dimension. Figure 3 illustrates how partial correlation coefficients represented the relation of ramus height to body length better than the simple correlation coefficients. The plotted simple correlation coefficients indicate that ramus height was not related to body length when the effects of the other variables were not considered. But, with the other variables held constant, the partial correlation coefficients plot illustrates an inverse relation between ramus height and body length at ages four and nine years.

The multiple regression technique permitted assessment of the regression of one predicted (x) variable on a combination of predictor (x) variables. Accordingly, Phase 4 was executed. However, before the regression equations were computed, the data were statistically screened for outliers, hence the data of two individuals were deleted to increase the precision of the regression equations (Dixon and Massey, 1957).

From the thirty multiple regression equations, only the ones that were statistically significant are shown in Table 4. Each of these predicts size of one mandibular dimension from a combination of the other four. The data for each of the twenty equations that were significant are listed. If a multiple regression equation uses a particular measurement, the coefficient by which the measurement must be multiplied will be found with the designated measurement. To illustrate, the equation to predict body length (x_2) at 9 years was

TABLE 4.

MULTIPLE REGRESSION EQUATIONS AND RESIDUAL ERRORS

x_1 = Ramus Height x_3 = Maximum Length
x_2 = Body Length x_4 = Bigonial Width

Age Group (Years)	EQUATIONS*	Residual Error
4	$x_1 = 0.336 - 0.573\, x_2 + 0.796\, x_3$	0.154
6	$x_1 = 1.127 - 0.432\, x_2 + 0.629\, x_3$	0.199
4	$x_2 = 0.235 - 0.314\, x_1 + 0.798\, x_3$	0.114
5	$x_2 = -0.895 + 0.782\, x_3$	0.136
6	$x_2 = -1.981 + 0.667\, x_3 + 0.292\, x_4$	0.148
7	$x_2 = -1.612 + 0.622\, x_3 + 0.301\, x_4$	0.149
8	$x_2 = -0.538 + 0.579\, x_3 + 0.223\, x_4$	0.164
9	$x_2 = -0.703 - 0.296\, x_1 + 0.672\, x_3 + 0.295\, x_4$	0.148
4	$x_3 = 2.011 + 0.464\, x_1 + 0.847\, x_2$	0.117
5	$x_3 = 4.362 + 0.797\, x_2$	0.137
6	$x_3 = 3.060 + 0.398\, x_1 + 0.747\, x_2$	0.158
7	$x_3 = 4.540 + 0.802\, x_2$	0.185
8	$x_3 = 5.220 + 0.721\, x_2$	0.190
9	$x_3 = 5.673 + 0.677\, x_2$	0.188
4	$x_4 = 2.625 + 0.712\, x_2$	0.280
5	$x_4 = 2.627 + 0.717\, x_2$	0.295
6	$x_4 = 3.651 + 0.581\, x_2$	0.287
7	$x_4 = 3.423 + 0.621\, x_2$	0.270
8	$x_4 = 3.968 + 0.557\, x_2$	0.303
9	$x_4 = 3\ 043 + 0.6$[illegible]$8\, x_2$	0.304

* Significant at 5% probability level; nonsignificant coefficients deleted.

selected. The equation from Table 4 is:

$$x_2 = -0.703 - 0.296\ x_1 + 0.672\ x_3 + 0.295\ x_4.$$

The measurements taken on child #8 from the files of the Child Study Clinic were: ramus height, 4.92 cm; maximum length, 10.78 cm; and bigonial width, 7.97 cm. These values are then entered into the multiple regression equation given above.

$$x_2 = -0.703 - 0.296\ (4.92) + 0.672\ (10.78) + 0.295\ (7.97) = 7.44\ \text{cm}$$

The predicted value, 7.44 cm, compares very closely with the actual measurement of body length, 7.47 cm, on this child.

Table 4 reveals that bigonial width was consistently predicted by the same variable at each age, whereas maximum length and body length were not predicted consistently by the same variables. Ramus height was not predicted for four of the age groups and bicondylar width was not predicted at any age. The residual error of all regression equations increased directly with age, meaning that the independent variables were less able to account for additional variation in any dependent variable as the mandible grew. Comparison of the residual errors in Table 4 and the standard deviation of the dependent variable in Table 1 indicates that practical use of these equations for prediction in the later ages of the study group will lead to estimates of lower precision.

Discussion

Traditional cephalometric investigations have contributed little to the understanding of interrelations within the facial complex. The face has been viewed as one flat area rather than a composite of individual three-dimensional bones. For example, Nanda (1955), Meredith (1960) and Harris (1962) described mandibular dimensions but did not correct the distortion in the two-dimensional lateral cephalometric image of the three-dimensional bone.

In this study, both the frontal and lateral cephalograms were utilized to derive five anatomically accurate measurements that represent the entire mandible. The results demonstrated that the mandible, which is one facial bone, was not interrelated in height, width and depth. Thus, dimensions of large mandibles were not all proportionally large, and dimensions of small mandibles were not all proportionally small. However, all dimensions except bicondylar width did vary with at least one other mandibular dimension. Washburn (1951) emphasized the importance of analyzing complexes within the human body that vary independently. He found that the regions of the coronoid process, dentition, main core, and gonial angle in the mandible varied independently. Horowitz and Thompson (1964) found that the chin varied independently of the rest of the mandible. In the present study, bicondylar width was detected as an additional dimension that varied independently. Ramus height was not related to body length when the effects of the other variables were not considered. However, when the other variables were held constant, there was an inverse relation demonstrated between ramus height and body length. For this reason the present finding is not in agreement with the findings of Morant (1936) who found that condylar process length was independent of body length. The implications of this inverse relation between mandibular segments may be better understood by some simple comparisons. If different sized mandibles of several girls are compared, many differences between mandibular segments may be seen. If, however, mandibles of the same maximum length, bigonial, and bicondylar widths are compared with each other, the comparison should indicate that when either ramus height or body length is proportionately large, the other is small.

The relation of mandibular depth and bigonial width is of particular interest, because the gonial area is common to both. Meredith (1963) found that the rate of growth of these two dimensions was positively related. This direct relationship, considered in the light of the antagonism of ramus height and body length, indicates that the gonial area determines the posterior and lateral enlargement of the mandibular body in contrast to the downward thrust of condylar growth.

The regression analysis demonstrated that the quantitative relationship decreased as the variation of the dimensions increased. This finding was not detected in the correlation analysis. In this study the regression analysis was conducted cross-sectionally at each age. Combinations of the variables were found to estimate all variables except bicondylar width. In a further investigation the longitudinal aspects of the data will be exploited to "lag" the *dependent* variable and derive equations that will utilize variables at one age to make predictions of future facial growth. For example, if size of ramus height at 9 years is used as the dependent variable, sizes of each mandibular dimension (i.e., ramus height, body length, maximum length, bigonial width and bicondylar width) at 4 years of age are used as "predictor" variables in the multiple regression equation. This analysis will provide an equation which can be used to predict growth of ramus height five years ahead.

Summary

Interrelations of five mandibular dimensions and prediction of each dimension from a combination of the other four were determined at each age on twenty-seven girls. Ramus height, body length, maximum length, bigonial width and bicondylar width were measured on posteroanterior and lateral cephalograms, and three-dimensionally corrected for radiographic enlargement and distortion. Characteristic statistics, simple and partial correlation coefficients, and multiple regression (prediction) equations were calculated.

Findings indicated significant correlations for maximum length to body length and to ramus height, and bigonial width to body length. Nonsignificant correlations were found for bigonial width to ramus height and to maximum length, and for bicondylar width with any dimension. Ramus height and body length were inversely correlated at four and nine years.

Regression analysis established that bigonial width was consistently predicted by the same variable. Maximum length and body length were predicted by several combinations of variables. Ramus height was not predicted at four ages, nor was bicondylar width at any age. Residual errors of regression equations increased directly with age.

The mandible, which is one facial bone, was not interrelated in height, width and depth. All dimensions except bicondylar width varied with one other dimension. The inverse relation of ramus height and body length indicates that if height or depth were large, the other would be small.

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